HURRICANE TRAJECTORY FORECASTS FROM A NON-DIVERGENT, NON-GEOSTROPHIC, BAROTROPIC MODEL

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ABSTRACT

A numerical method for obtaining trajectories from non-divergent, non-geostrophic, barotropic forecasts is presented. The wind used in the computation of direction and speed of movement is derived from a Least Squares cubic surface fitted to sixteen stream-function values surrounding the point of interest. One-hour time steps are used. The method is applied to the forecasting of hurricane tracks and useful results are obtained for periods up to 36 hours. Several other uses of trajectory forecasts are suggested.

1. INTRODUCTION

An increasing number of meteorologists, and others, have become interested in obtaining more accurate knowledge of the trajectories of air particles. Some are primarily concerned with determining the paths which particles followed in arriving over a particular location—in short, they are interested in hindcasts. Others are more concerned about knowing the path a particle (or any other conservative element) will take for the next few hours or days. These people are interested in forecast trajectories.

For two years, the Joint Numerical Weather Prediction Unit (JNWP) has been making numerical forecasts on a daily basis using the IBM model 701 electronic computer. One of the forecasts currently being made is based on a non-divergent, non-geostrophic barotropic model (see e. g., Bolin [1]) using stream-function values as initial data. These input values are obtained from a solution of the "balance equation" given by Shuman [2].

During the progress of the 500-mb. barotropic forecast, the stream-function values (scaled as heights) for a 30 $\times$ 34 grid are stored on a "history tape." The mesh length of the grid is approximately 180 nautical miles at latitude 60°, and the area covered is roughly two-thirds of the Northern Hemisphere. Since a time interval of 1 hour is used and the forecast is carried out to 3 days, 72 fields of stream-function values are available on this tape.

The fact that the barotropic forecasts have been quite successful suggested that a numerical method for obtaining forecast trajectories might be devised as a useful and relatively inexpensive by-product. In particular, the novelty of obtaining non-divergent, non-geostrophic trajectories based on 1-hour time steps seemed very attractive.

The general purposes of this paper are to describe the method, show the results from one major application, and point out other possible uses. In particular, tests of forecast tracks for several hurricanes from 1955 and 1956 are presented and discussed in some detail. An attempt has been made to draw certain conclusions from these cases.

2. METHOD

The method used here is based on the fitting of a third-order polynomial to sets of forecast stream-function values surrounding the point of interest (p). The polynomial can be expressed in the form:

$$Z=A+Bx+Cy+Dx^2+Ey^2+Fxy+Gx^3+Hy^3+Ix^2y+Jxy^2 \tag{1}$$

In order to determine the ten coefficients in (1) we obviously need to know the 500-mb. stream-function values at ten grid points. Actually the polynomial is fitted to sixteen values at grid points distributed almost symmetrically about the point p using the method of Least Squares. The origin is arbitrarily taken at the grid point forming the lower left corner of the square in which p is located at any time step. The grid layout is shown in figure 1. As the point in question moves through the basic forecast grid (i, j), the floating, sixteen-point grid (x, y) is shifted accordingly.

The coefficients in (1) are first determined by solving sets of ten, simultaneous, algebraic equations using the 500-mb. data at the sixteen surrounding points. The first derivative of (1) with respect to both x and y is then evaluated at the known x, y position. These derivatives are clearly proportional to the x- and y-components of

*Any opinions expressed by the author are his own and do not reflect the views of the Navy Department at large.

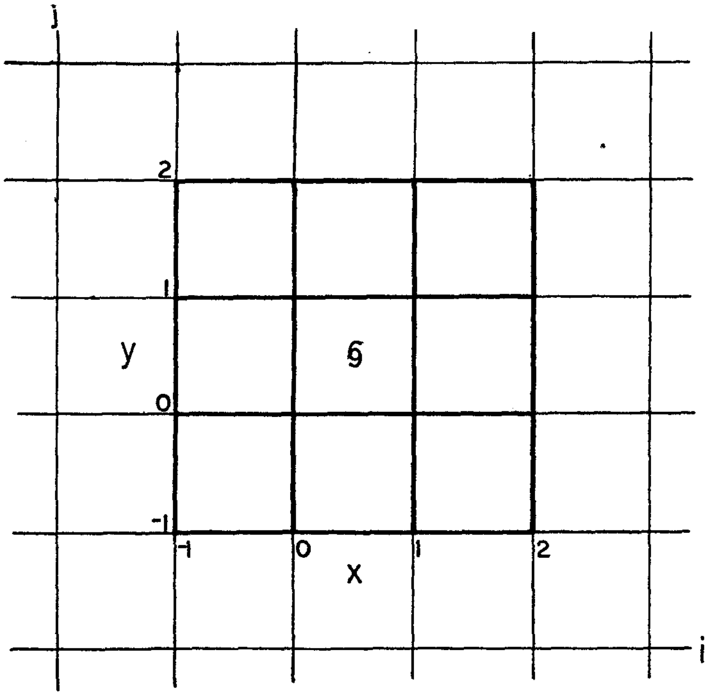

FIGURE 1.—Grid layout for cubic method. Floating 16-point grid (x, y) superimposed on basic, barotropic grid (i, j). One mesh length is approximately 180 nautical miles at latitude 60°.

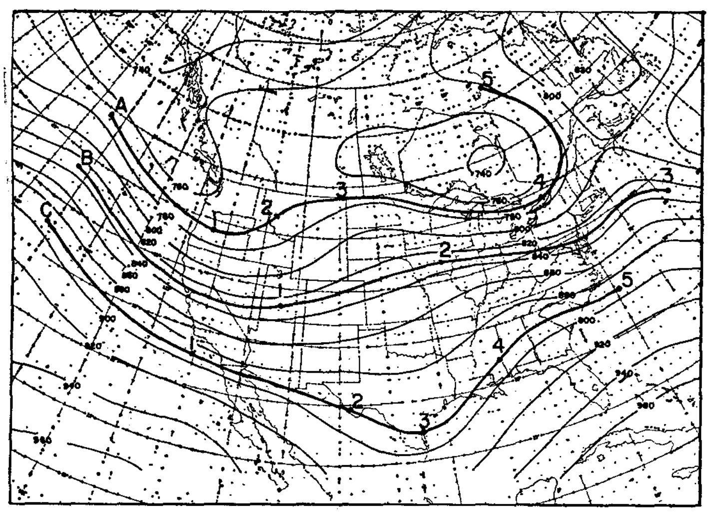

FIGURE 2.—Trajectories of three points (A, B, C) obtained by advection in a constant (with time) stream-function field for 0300 GMT, March 13, 1957. Thin solid lines, 500-mb. stream-function values scaled as heights; heavy solid lines, trajectories from cubic code; heavy numerals, number of days from beginning of each trajectory.

the wind at point p (in the cubic surface). Given the initial i and j coordinates of p, its position at any later instant can be determined from the following:

$$i_{t+\Delta t}=i_t-\frac{gm^2}{\bar{f}d^2}\frac{\partial Z}{\partial y}\Delta t \tag{2}$$

$$j_{t+\Delta t}=j_t+\frac{gm^2}{\bar{f}d^2}\frac{\partial Z}{\partial x}\Delta t \tag{3}$$

where g is the gravitational acceleration, m is the map factor, $\bar{f}$ is the Coriolis parameter at latitude 45° (used in scaling the stream-function), d is the mesh length, t is time, and Z is the 500-mb. stream-function value.

As a test of the method, three points were advected in a constant (with time) stream-function field for 0300 GMT, March 13, 1957. The results of this experiment are shown in figure 2. The greatest deviation from the initial stream-function value occurred at the end of five days in trajectory A and amounted to a maximum of only 90 feet. The cubic surface is evidently of sufficiently high order to yield good results when used to compute winds. The use of non-centered differences for hourly intervals is also quite satisfactory. Furthermore, examination of stream-function charts versus the conventional constant-pressure charts used in most meteorological agencies reveals that observed winds more closely follow the directions indicated by the former—as one should expect.

3. FORECASTING HURRICANE TRACKS

In general, tropical storms are steered by the tropospheric current in which they are embedded. It has been long recognized that the flow at 500 mb. is fairly representative of the mean flow in the troposphere, and that hurricanes do tend to move with a direction and speed close to that of the basic current around them at this level.

Several techniques for forecasting hurricane motion up to 24 hours in the future have evolved in recent years. The method of Riehl and Haggard [3] has been widely tested in this country, and a numerical method developed by Sasaki and Miyakoda [4] is being used by the Japanese in the Pacific area. They have, however, been forced to use graphical methods of solution for lack of a high-speed electronic computer.

The success of these approaches suggested that the trajectory method described here might lend itself to the forecasting of hurricane movement. One would merely replace the storm circulation with a point vortex and steer it using the forecast flow at the 500-mb. level. Four Atlantic hurricanes from the 1955 and 1956 seasons were selected for study.

In figure 3 is shown a 48-hour forecast track for hurricane Flossy starting at 1500 GMT on September 23, 1956. In this particular case Flossy's circulation was subtracted from the initial 500-mb. chart by subjective smoothing before the barotropic forecast was started. Figure 3 shows that the predicted and observed tracks agreed remarkably well for the first 36 hours. After that time, the tracking code failed to catch the observed deceleration and curvature toward the northeast. Instead, a rather

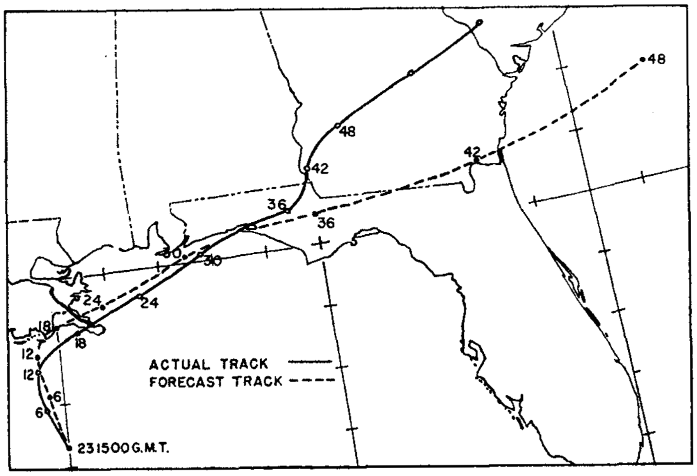

FIGURE 3.—Numerical test for hurricane Flossy starting at 1500 GMT, September 23, 1956. Heavy solid line, observed track; dashed line, forecast track; numerals indicate forecast and observed positions at various hours. Storm circulation subtracted from initial flow pattern.

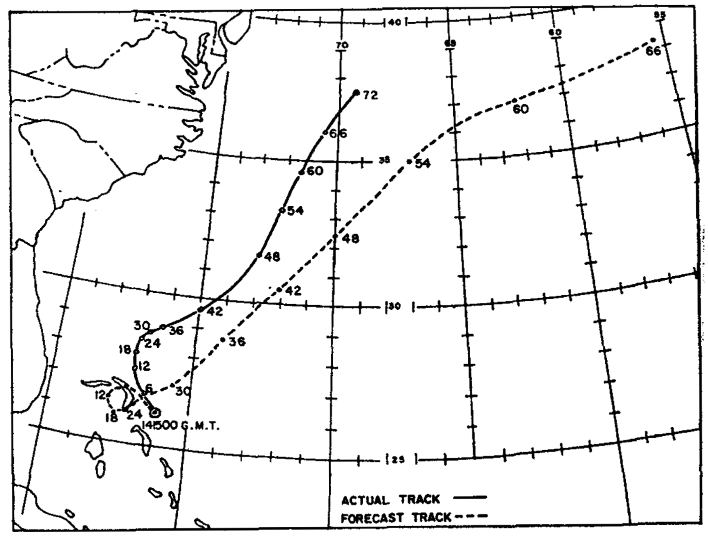

FIGURE 4.—Numerical test for hurricane Betsy starting at 1500 GMT, August 14, 1956. Storm circulation left in initial flow pattern.

even acceleration toward the east-northeast was forecast after the recurvature had been completed.

Figure 4 shows the results of a numerical test on hurricane Betsy of 1956 starting at 1500 GMT August 14. In this case the storm circulation was not removed from the 500-mb. input data. The counterclockwise loop which was forecast during the first 24 hours resulted from the initial i and j in the tracking system being located slightly to the north side of the circulation center as defined by the cubic surface. By 30 hours, however, the closed circulation of the tropical system had disappeared from the barotropically-forecast flow pattern due to the finite-difference equations and smoothing used in the forecast model, and a more regular path resulted. In spite of the initial loop the forecast was, in general, rather good, for there was some doubt on the 14th whether the hurricane would continue toward the coast or begin to recurve. Once again too much acceleration was forecast after the storm had become embedded in the westerlies.

The third hurricane which was selected for numerical tracking was Connie of August 1955. This storm was small in scale and demonstrated a very erratic movement throughout most of its life history. When the hurricane circulation was subtracted from the initial chart, it was found that very little steering current remained in the vicinity of the center. For this case three different fractions (60, 80, and 100 percent) of the 500-mb. wind were used to advect the point vortex. The resulting paths obtained for the 60 and 100 percent values (fig. 5) show that a col must have developed in the forecast basic flow immediately surrounding the storm center. The differences in the forecast tracks which resulted from a 40 percent reduction in the wind clearly demonstrate the weakness of any steering technique. Once again we are confronted with the same dilemma which has plagued

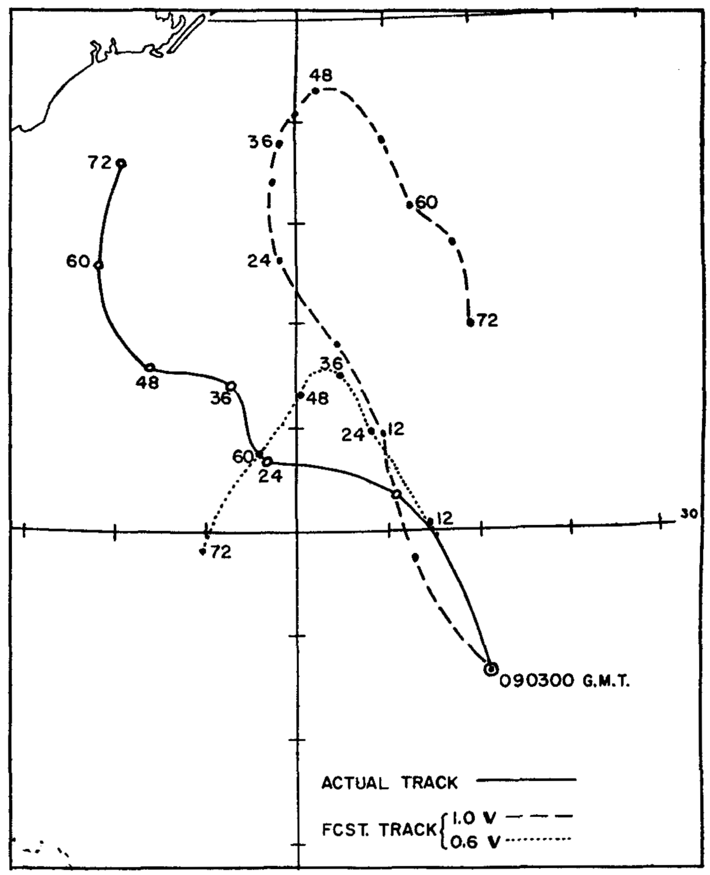

FIGURE 5.—Numerical test for hurricane Connie starting 0300 GMT, August 9, 1955. Heavy solid line, observed track; dashed line, forecast track using 100 percent of 500-mb. wind; dotted line, forecast track using 60 percent of 500-mb. wind; numerals indicate forecast and observed positions at various hours. Storm circulation subtracted from initial flow pattern.

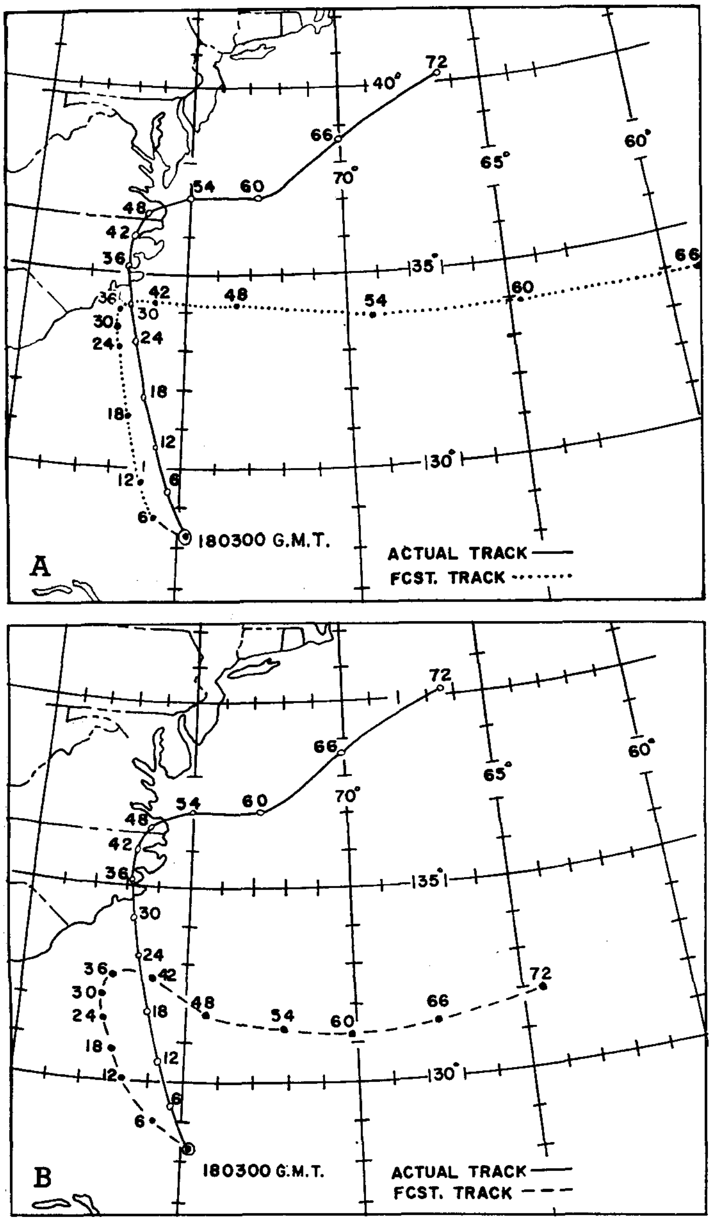

FIGURE 6.—Numerical test for hurricane Ione starting 0300 GMT, September 18, 1955. (A) Storm circulation left in initial flow pattern; (B) Storm circulation subtracted from initial flow pattern.

hurricane forecasters for years; namely, choosing the proper steering level before starting the "prog" instead of after the fact.

Two numerical runs were made for hurricane Ione starting at 0300 GMT, September 18, 1955. In one, the storm circulation was subtracted from the total flow pattern before the barotropic forecast was started; in the other, the circulation was left intact. The results of these two runs (fig. 6) show that the forecast which included the hurricane circulation was clearly superior. The northward component of movement was not great enough in either forecast; however, the rapid curvature toward the east occurred at approximately the right time.

Obviously, the substitution of a point vortex for the storm circulation eliminates any possibility of interaction between the hurricane and the steering current. According to Kasahara [5], this omission introduces an error in the movement whose magnitude is dependent upon the scale of the vortex and the absolute vorticity gradient of the basic flow. He makes a correction in the forecast movement by allowing the basic current and a circular vortex held constant with time to interact with each other. The relatively few cases tested at JNWP do indicate that the results are better if one does not subtract the storm circulation from the total current before starting the barotropic forecast. In the mean, the errors resulting from the simple method described here appear to be no larger than those reported by Kasahara. A summary of these errors is presented in table 1.

TABLE 1.—*Vector error between forecast and observed position, in nautical miles. In "smoothed" runs, the storm circulation was subtracted from the initial data by subjective smoothing; in "unsmoothed" runs, the total flow pattern was left intact.*

Storm	Wind factor	Type	Number of hours					
			12	24	36	48	60	72
Flossy	1.0	Smoothed	15	40	30	300	-----	-----
Betsy	1.0	Unsmoothed	50	120	120	155	460	-----
Connie	0.6	Smoothed	25	60	60	90	120	220
Connie	0.8	Smoothed	20	80	100	100	110	200
Connie	1.0	Smoothed	40	120	150	185	180	225
Ione	1.0	Smoothed	60	100	150	300	360	490
Ione	1.0	Unsmoothed	60	25	70	190	440	-----

4. CONCLUSIONS

Assuming that one is really justified to draw conclusions from a few cases, the following are offered for discussion:

(1) Winds computed in the cubic surface are of sufficient accuracy for the trajectory problem.

(2) Useful forecasts of hurricane movement may be obtained from the trajectory method in most cases, even though the prognostic tracks are far from perfect.

(3) The interaction between the hurricane and its surrounding current must be allowed for in some manner. Leaving the storm circulation in the initial flow pattern appears to improve the forecast; however, the circulation is usually "swallowed up" during the progress of numerical iteration.

(4) The 500-mb. level is not necessarily representative of the mean steering current for all hurricanes. Additional tests with different levels as well as integrated (vertically) flow charts for initial conditions are being run at JNWP. For small storms a lower level appears to be more appropriate, while a higher level seems to be indicated for larger storms.

Other members of the JNWP Unit are currently using this trajectory method to study conservation of vorticity, deformation, and other problems. In addition, the same method, or some variation thereof, could undoubtedly prove suitable for forecasting balloon trajectories, fall-out patterns, moisture trajectories, etc. It is planned to code this problem for the new IBM 704 which the JNWP Unit will install in July of 1957.

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